

EECS 452 – Lecture 24

Today: Sigma-Delta D/A converter
 Cascaded integrator comb filter usage

Handouts: printed copy of today's lecture slides

Read: about DSP!

References:

Last one out should close the lab door!!!!

Please keep the lab clean and organized.

It is... fruitless to question and debate early design decisions; better solutions are often quite obvious in hindsight. Perhaps the most important point was that someone did make decisions, in spite of uncertainties. - Niklaus Wirth

Overview

- ▶ Basic D/A conversion.
- ▶ Mathematics of reconstruction.
- ▶ Zero-order hold and its transfer function.
- ▶ Inserting zeros between samples: up-sampling.
- ▶ Zero-order hold effects on up-sampled spectrum.
- ▶ Sigma-delta D/A conversion.
- ▶ Cascaded integrator comb filter.

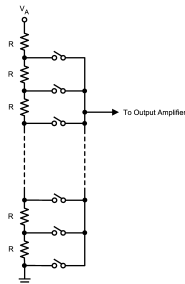
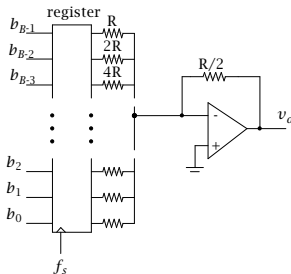
The key to Sigma-Delta conversion

The signal and the noise enter the system at different points. With care, this allows filtering the noise while not affecting the signal.

The noise is not moved. It is filtered. The filtering is designed to minimize the in-band noise at the expense of the out-of-band noise. The out-of-band noise is then filtered (attenuated).

We saw this with the sigma-delta A/D last time.

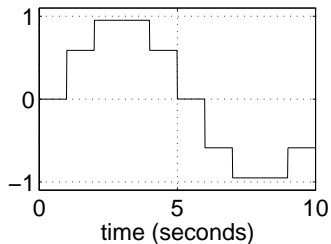
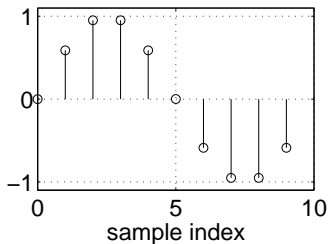
Going from numbers to analog



From DAC121S101 data sheet.

Two of many possible methods of implementation are shown.
Digital input value is held constant between clock pulses.
Output waveform is has stair step appearance.
Termed, using a *zero-order hold*.

The zero-order hold



Sample value weighted reconstruction

“pulses”

D/A conversion can be modeled by taking a weighted set of reconstruction pulses and forming a continuous time waveform.

$$\begin{aligned}y(t) &= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot h(t - nT_s) \\&= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s) \otimes h(t)\end{aligned}$$

where $h(t)$ is the reconstruction pulse (e.g., a square pulse of height 1 and width T_s). Taking the Fourier transform:

$$Y(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - \frac{n}{T_s}) \cdot H(f)$$

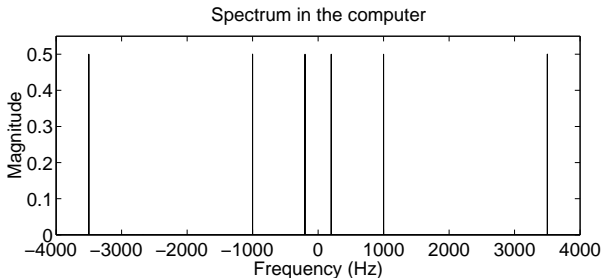
Zero order hold transfer function

The zero order hold corresponds to use of a pulse which has unit amplitude over the interval $[0, T_s]$, $T_s = 1/f_s$, and zero elsewhere.

$$H(f) = \int_0^{T_s} e^{-j2\pi ft} dt = \frac{e^{-j2\pi f/f_s} - 1}{-j2\pi f} = e^{-j\pi f/f_s} \frac{\sin(\pi f/f_s)}{\pi f}$$

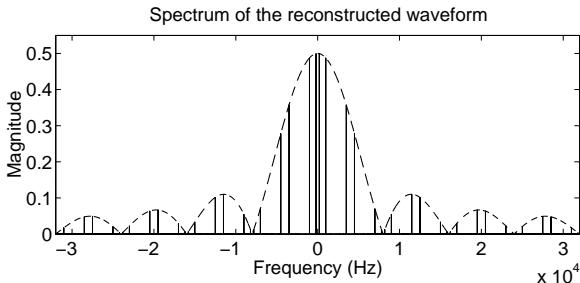
The case where $f = 0$ is easily handled by noting that the value of the integral for this case is $1/f_s$.

Simulation waveform spectrum



The spectrum of the sum of three unit amplitude sine waves. The sample rate, f_s was 8000 Hz and the sine frequencies were 200 Hz, 1000 Hz, and 3500 Hz.

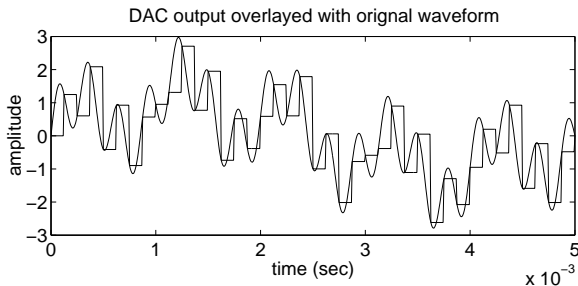
Samples directly to D/A with zero-order hold



Spectrum using a zero-order hold D/A converter. The dashed line shows the effects of the zero-hold on the spectrum magnitude.

The D/A output waveform possesses images of the baseband spectrum. These occur at multiples of the conversion rate f_s . The use of the zero-hold places an amplitude weighting of $\sin(\pi f / f_s) / (\pi f / f_s)$ on the spectrum at the D/A output.

Original and reconstructed waveform



Zero filled waveform DFT

Consider a waveform $x(t)$ sampled at rate f_r to obtain $N = f_r T$ samples. Between each sample insert $M - 1$ zero values. The resulting set of samples now contains MN values.

The DFT of the MN value data can be related to the DFT of the original N value data set.

Write the set of samples obtained using sample rate f_r as $x[n]$ where $0 \leq n < N$. Construct a zero filled set $x_M[r]$ such that $x_M[r] = x[n]$ for $r = Mn$ where $0 \leq n < N$ and zero otherwise.

The DFT of $x_M[r]$ is (using the $1/N$ scaling convention)

$$X_M[k] = \frac{1}{MN} \sum_{r=0}^{MN-1} x_M[r] W_{MN}^{kr}$$

where $0 \leq k < MN$ and $W_N = e^{-j2\pi/N}$.

Zero filled waveform DFT (continued)

Write $r = Mn + m$ where $0 \leq n < N$ and $0 \leq m < M$. Then

$$X_M[k] = \frac{1}{MN} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} x[Mn + m] W_{MN}^{k(Mn+m)}.$$

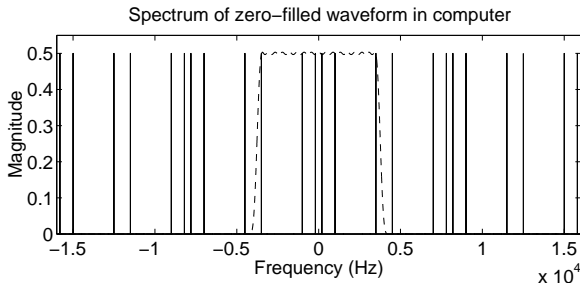
Because $x_M[Mn + m] = 0$ for $m \neq 0$ and $x_M[Mn] = x[n]$ we have

$$X_M[k] = \frac{1}{MN} \sum_{n=0}^{N-1} x_M[Mn] W_{MN}^{kMn} = \frac{1}{M} \left(\frac{1}{N} \sum_{n=0}^{N-1} x[n] W_N^{kn} \right)$$

where $0 \leq k < MN$. Note the $1/M$ factor.

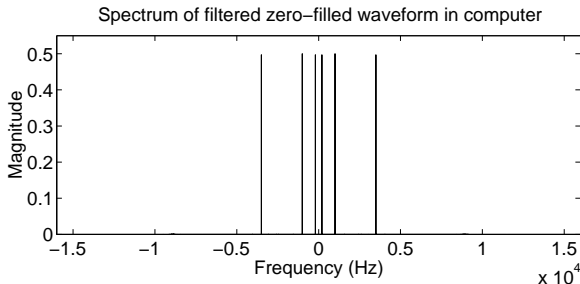
Zero filling in this manner causes the N value DFT of $x[n]$ to repeat M times to make up the MN value DFT of the zero filled set of samples.

4x upsampled waveform spectrum



The DFT of the values in the computer after inserting three zero's between each original sample. The dashed line shows the filtering that is going to be applied in the computer prior to D/A conversion.

Filtered 4x upsampled waveform spectrum



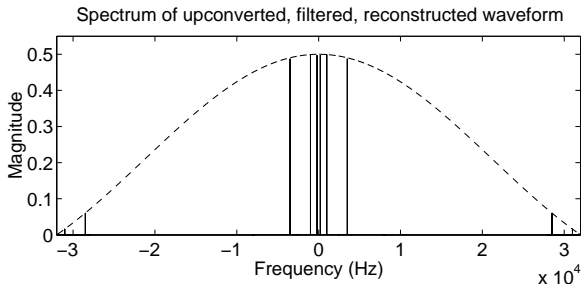
Digitally lowpass filtered spectrum of the zero-filled waveform prior to being sent to the D/A converter at rate $4f_r$.

Short summary of upsampling

The effective sample rate is increased by a factor of M by creating a new time series by inserting $M - 1$ zero values between values at rate f_r and then digitally filtering the result. This is referred to as up-conversion of the sample rate by a factor of M .

A benefit of the up-conversion process is that the quantization noise caused by using a finite word length is spread over broader frequency range. With proper analog filtering a D/A converter using fewer bits but a higher construction rate can perform as well as a converter using more bits but at a lower rate.

Zero order hold effect on oversampled reconstruction



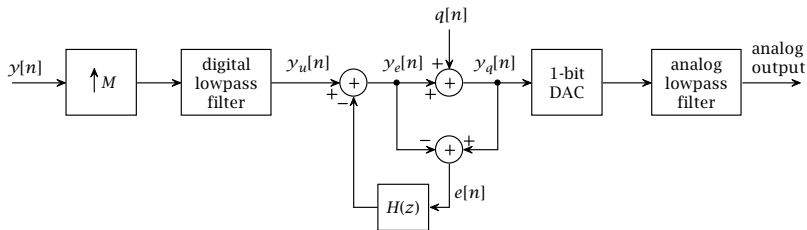
Spectrum of the D/A converted upshifted and low pass filtered waveform. Dashed line illustrates the effect of the zero-hold on the output spectrum.

Often pre-correct for $\sin(x)/x$ weighting

We can correct for the $\sin(f)/f$ weighting in the computer prior to sending samples to the D/A effectively flattening the spectrum.

There are still undesired frequency components present at the DAC output. These are at much higher frequencies than prior to up conversion and at lower amplitude. These are relatively easily removed using an analog filter at the DAC output.

Sigma-delta D/A converter block diagram



Input to output TF

$$Y_e(z) = Y_u(z) - H(z)E(z)$$

$$Y_q(z) = Y_e(z)$$

$$E(z) = Y_q(z) - Y_e(z)$$

From this set of equations it is seen that the transfer function between $Y_u(z)$ and $Y_q(z)$ equals 1 in the absence of noise.

$$Y_q(z) = Q(z) + Y_e(z)$$

$$E(z) = Y_q(z) - Y_e(z)$$

$$Y_e(z) = -H(z)E(z)$$

Quantization noise to output TF

Solving

$$\begin{aligned}E(z) &= -\frac{Y_e(z)}{H(z)} \\Y_e(z) &= \frac{H(z)Y_q(z)}{H(z) - 1} \\W(z) &= \frac{Y_q(z)}{Q(z)} = 1 - H(z)\end{aligned}$$

In many texts $H(z) = z^{-1}$. Not sure that this is what is used in practice. For this $H(z)$ we have

$$W(f) = 1 - e^{-j2\pi f/f_s}$$

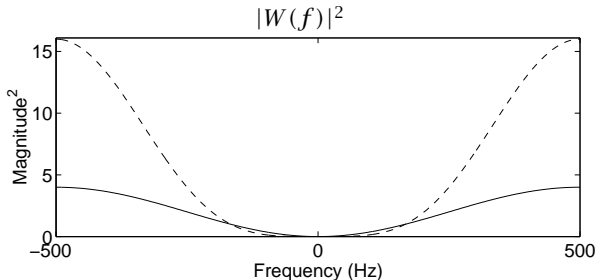
giving

$$|W(f)|^2 = 4 \sin^2(\pi f/f_s).$$

Two useful $H(z)$

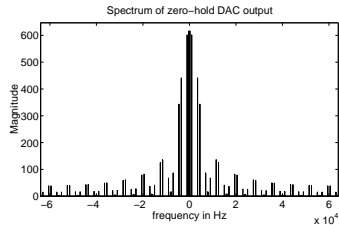
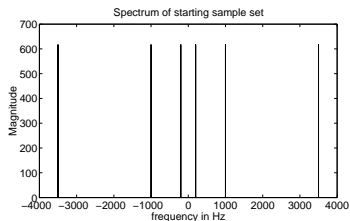
The system performance can be improved by replacing the single z^{-1} stage by more sophisticated filter. A filter that has a high pass transfer function will provide improved performance. One example is the transfer function

$$z^{-1}(2 - z^{-1}).$$



Solid line is for $H(z) = z^{-1}$. Dashed line is for $H(z) = z^{-1}(2 - z^{-1})$.
 $f_r = 1000$.

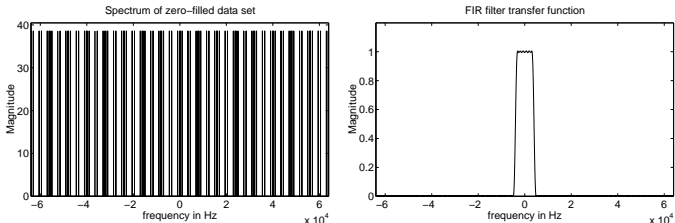
Simulation input and zero-order hold spectra



$f_s = 8000$
line amplitudes,

$T = 16$ seconds

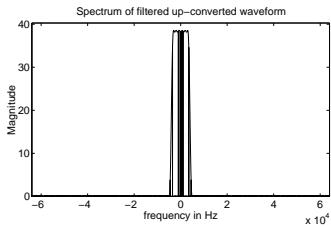
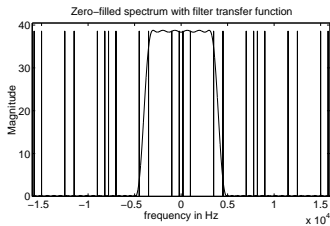
Spectra of zero filled data and FIR filter TF



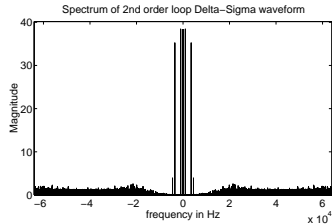
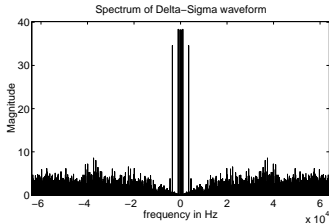
Filter designed using MATLAB's `remez` function,
`B=REMEZ(N,F,A)`.

```
fset=[0 0.8/M 1.2/M 1];  
amps=[1 1 0 0];  
b=remez(181, fset, amps);
```

Filter TF and the upsampled waveform spectrum



One-bit spectra, first and second order filters



MATLAB simulation used 16 times oversampling.

The DSP elevator

A very common DSP activity is sample rate conversion.

Sometimes the change is up and sometimes it's down.

Sigma-Delta A/D converters start high and end up (relatively) low. Sample fast with few bits, do sigma-delta, filter to interpolate then discard samples (decimate).

Sigma-Delta D/A converters start low and end up (relatively) high. Interleave zeros, do sigma-delta, filter to eliminate images use fast D/A and finally use low cost analog filter.

Communications systems make extensive use of up/down sample rate conversion.

Filters for large factor sample rate conversion

Filter bandwidth small relative to sample rate. Difficult to achieve.

Commonly used filter technologies include:

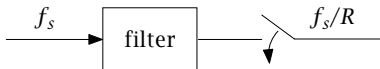
- ▶ “standard” FIR filters,
- ▶ stage rate change using half-band filters,
- ▶ coefficient slicing designs,
- ▶ polyphase filters,
- ▶ cascaded-integrator-comb (CIC) filters,

We will take a close look at the CIC filter.

E.B. Hogenauer, *An Economical Class of Digital Filters for Decimation and Interpolation*, IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-29, No. 2, April 1981.

fredric j harris, *Multirate Signal Processing for Communications Systems*, Prentice Hall, 2006.

First, a look at down sampling

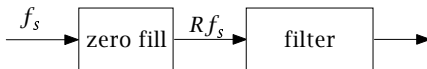


Assume use of a FIR filter. Why compute the samples that are not used?

Next, consider up sampling

For an up-sampling factor of R insert $R - 1$ zeros between the original sample values.

Again assume use of a FIR filter. Why form the products of the coefficients and the zero values?

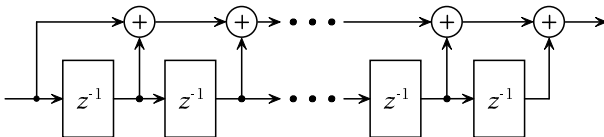


The box car (unit weight) FIR filter

Consider the length RM FIR filter whose coefficient values are all equal to one.

$$y[n] = x[n] + x[n-1] + x[n-2] + \cdots + x[n-RM+2] + x[n-RM+1]$$

$$H(z) = 1 + z^{-1} + z^{-2} + \cdots + z^{-(RM-2)} + z^{-(RM-1)}$$



Boxcar filter transfer function

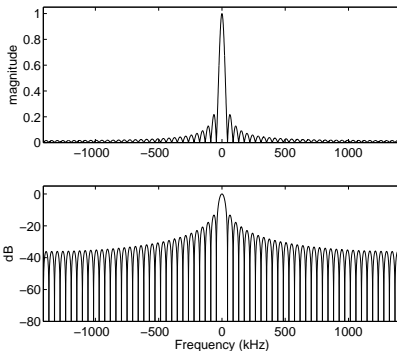
The transfer function is a geometric series and can be written in closed form:

$$H(z) = \frac{1}{RM} \frac{1 - z^{-RM}}{1 - z^{-1}}.$$

The $1/(RM)$ factor is added in order to normalize the filter DC gain.

Magnitude and dB plots for $H(z)$ for a sample rate of $44.1 \times RM$ kHz with RM equal to 64.

Not a very great filter.



$H(z)$ in terms of frequency

In terms of frequency $H(z)$ is

$$H(f) = e^{-j\pi(RM-1)f/f_s} \frac{\sin(\pi RM f/f_s)}{\sin(\pi f/f_s)}$$

where $RMf_s/2 \leq f < RMf_s/2$.

The value at $H(f)$ at $f = 0$ is RM .

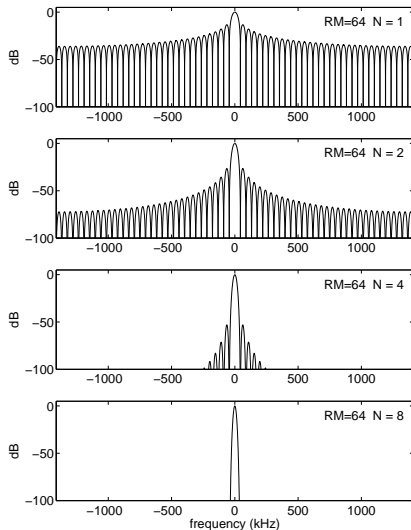
Zeros of $H(f)$ occur at $\pi RM f/f_s = \pm P\pi$ where P is integer.

Zeros of $H(f)$ occur at f equal to non-zero multiples of f_s/RM .

Typically $M = 1$ or $M = 2$.

Boxcar cascade transfer function

If we cascade several stages of identical filters we will push down the sidelobes and sharpen up the nulls. This figure shows the resulting transfer functions using a cascade of 1, 2, 4 and 8 identical filters.



Design freedom

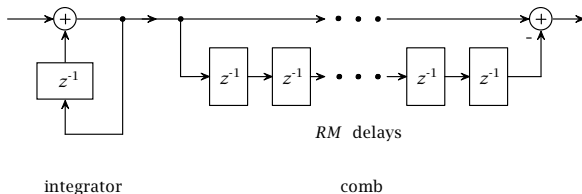
- ▶ M typically 1 or 2. Affects zero spacing.
- ▶ R relates to sample rate change factor.
- ▶ N number of filters in cascade. Increases fall off.

Gain at 0 Hz is RM . Need to scale value carefully. Note that we've normalized filter transfer function plots at 0 Hz to unity. This may not be true in practice.

Redraw block diagram

$H(z)$ can be thought of as being cascade of two filters. One filter corresponds to numerator and the other corresponds to the denominator,

$$H(z) = \frac{1}{RM} \left(\frac{1}{1 - z^{-1}} \right) (1 - z^{-RM}).$$

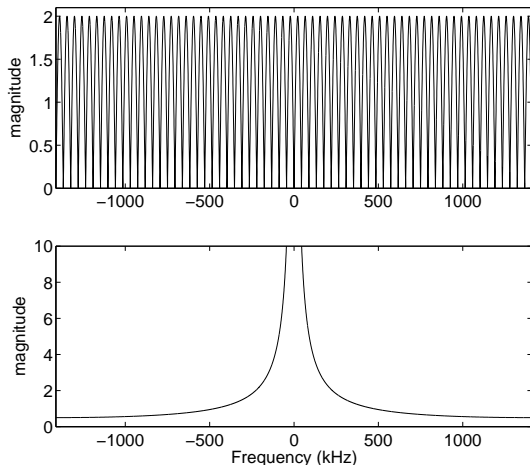


Changing the order of the blocks does not affect the cascade's transfer function.

Individual transfer functions

The numerator filter is called a *comb filter*. The reason for this name should be apparent looking at top plot which shows the magnitude of numerator transfer function.

The bottom plot shows the denominator transfer function (integrator). As expected it becomes quite large near zero frequency.



What does this gain us?

The original FIR filter contained $RM - 1$ delays and required RM additions per input/output sample.

The alternate form contains $RM + 1$ delays and requires only 2 additions per input/output sample.

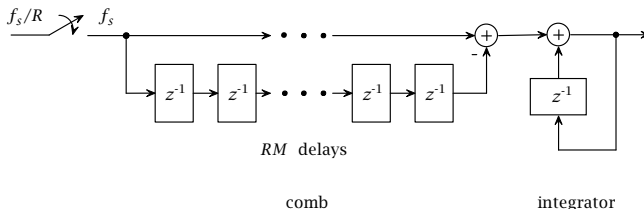
But won't the integrator overflow?

It very likely will!

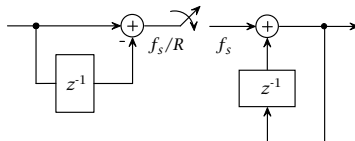
The problem is mitigated, if not totally eliminated, by the use of two's complement arithmetic. The integrator word size is to be determined by the number of bits used by the input and the value of RM . The integrator-comb has exactly the same transfer function as the unit weight FIR filter.

As long as a sufficient number of bits is used to give a correct end result, two's complement arithmetic doesn't care whether or not there were any intermediate overflows.

Up-sampling and interpolation

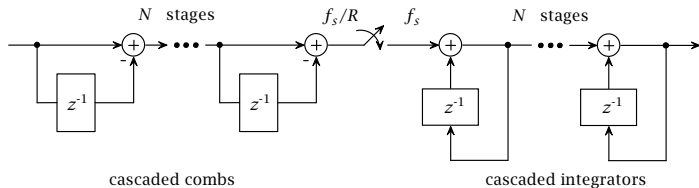


Set $M = 1$. The up-sampling consists of placing $R - 1$ zero values between the rate f_s/R samples. The up-sampler can be moved through the comb filter to give:



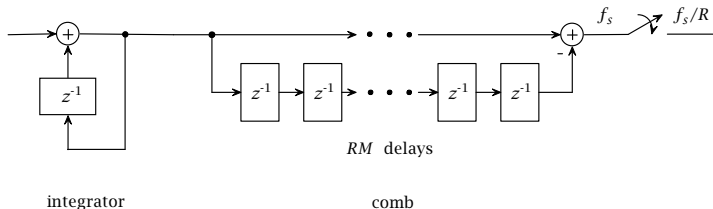
Up-sampled cascade

This works as well for a cascade of N stages. The block diagram for N stages is:

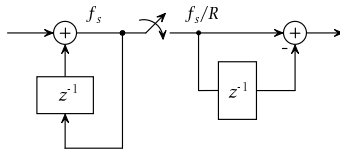


For $N = 8$ and $R = 64$ we originally would have required 504 delay stages and added 512 values per output value. The above block diagram gives numerically exactly the same output values and uses only 8 delay stages and adds only 16 numbers per output value.

Down-sampling and decimation

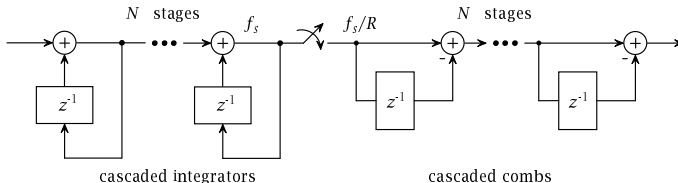


Set $M = 1$. The down-sampling consists of discarding $R - 1$ values between the rate f_s/R samples. The down-sampler can be moved through the comb filter to give:



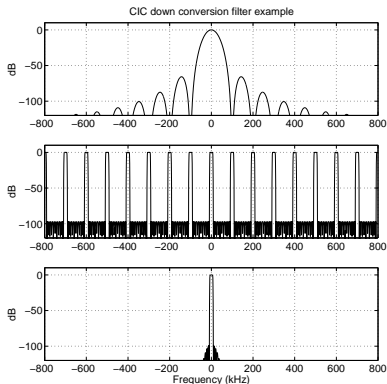
Down sampled cascade

This works as well for the cascade of N stages. The block diagram for N stages is:



Simulated HSP50016 model result

Modeled CIC spectra. Top is the output of the CIC filter prior to down sampling. Middle shows the replicated FIR filter transfer function. Bottom shows the components with the weightings that they will have after being aliased.



$F_s = 1600$ kHz
 $R = 16$
 $N = 5$
 $M = 1$
 $F_c = 6.3$ kHz

Comments on CIC

Advantages:

No multipliers are needed.

No storage is required for coefficients.

Storage is minimized by high rate integration, low rate differencing.

Regular architecture.

Minimal control structure required.

Same structure can be used for wide range of R values.

Concerns:

Integrator overflow not a problem with two's complement numbers.

Bit growth in decimators.

From Hogenauer, *An Economical Class of Digital Filters for Decimation and Interpolation*, IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-29, No. 2, April 1981.

Additional comments on CIC filters

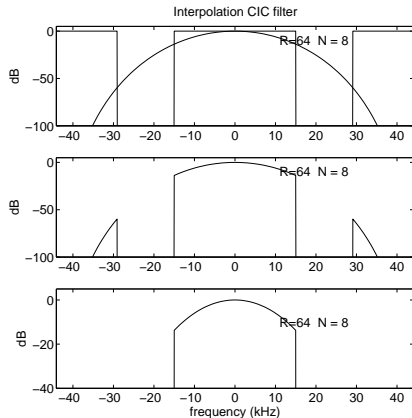
Cascade a (reasonably sized) FIR filter with the CIC. Use the FIR filter to convert the sample rate by a factor of 2 to 4. Do the bulk of the up/down sample filtering in the CIC.

This approach widens the main lobe of the CIC. The FIR filter, whose passband is more easily controlled, does the major portion of the filtering work around 0 Hz. The CIC does the bulk of the attenuation at the higher frequencies. An excellent combination.

The price on the TI GC5016 is \$65 in quantities of 1000. It is targeted at use in wireless base stations.

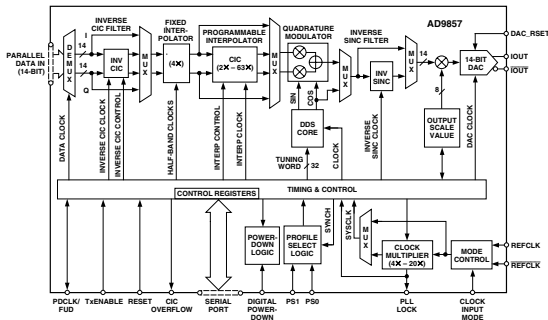
Up-conversion example

Fine detail for “brickwall” 15 kHz input spectrum with interleaved zeros. Top plot shows images and the CIC transfer function. Middle plot shows effect of the CIC on the input spectrum. Bottom plot better shows the shading over the baseband. A FIR filter can be included in the cascade to flatten this out.



AD9857 digital up converter

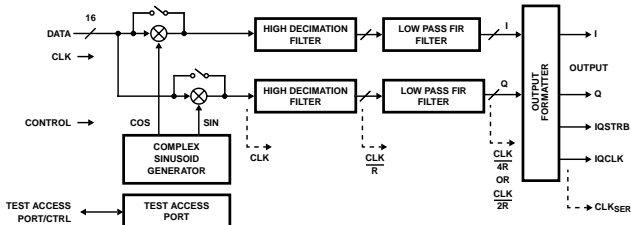
The Analog Devices AD9857 quadrature digital upconverter uses CIC filtering. Below is shown the functional block diagram of this part. The AD9857 is used to go from baseband to RF. Using a direct digital oscillator it can generate carrier frequencies up to 80 MHz.



Extracted from Analog Devices data sheet.

HSP50016 digital down converter

A CIC filter is used for sample rate reduction in Harris/Intersil digital down shifter HSP50016. Below is a functional block diagram of this part.



Extracted from Intersil data sheet.

HSP50016 digital down converter (cont.)

This part makes excellent use of the CIC concept. The CIC filter used in the HSP50016 is followed by a FIR to complete the filtering process. The FIR filter used in the part uses 131 coefficient values. My simulation (to follow) used a filter I manually attempted to match the HSP50016's filter. I ended up with 137 coefficients. My design also differs in that I did not compensate for the $\sin(\pi MRf) / \sin(\pi f)$ filter weighting in the pass band whereas the HSP50016 does.

Manufacturers using CIC filters

Primary use is for communication system RF support.

- ▶ Intersil
- ▶ TI
- ▶ Analog Devices
- ▶ XILINK
- ▶ others

The TI GC5016 down/up converter



GC5016

SLWS142B – JANUARY 2003 – REVISED SEPTEMBER 2003

WIDEBAND QUAD DIGITAL DOWN-CONVERTER/UP-CONVERTER

FEATURES

- Four Independently Configurable Wideband Down-Converter or Up-Converter Channels
 - Four Channel Down Convert Mode
 - Four Channel Up Convert Mode
 - Two Channels Down and Two Channels Up Mode
- Down-Conversion Channel Mode
 - Input Rates to 150-MSPS for Four Channels, 300-MSPS for Two Channels in Double Rate Mode
 - Four Wideband Down-Conversion Channels Support UMTS Standards
 - 115-dB SFDR
 - FIR Filter Block Consists of 16 Cells That Provide Up to 256 Taps Per Channel
 - 64 Parallel Input Bits and 64 Parallel Output Bits Provide Flexible I/O Options
 - Many Multiplex Output Options
- Up-Conversion Channel Mode
 - Output Rates to 150-MSPS for Four Channels, 300-MSPS for Two Channels
 - Four Up-Conversion Channels Support UMTS Standards
- FIR Filter Block Consists of 16 Cells That Provide up to 256 Taps Per Channel
- 64 Parallel Input Bits and 64 Parallel Output Bits Provide Flexible I/O Options
- Multiple Real and Complex Output O
- Two Channel Double Rate Real Output Mode With Rates to 300 MSPS
- Outputs Can Be Independent, Summed Into Two or One Output(s), and Optionally Merged With Multiple GC5016 Chips
- JTAG Boundary Scan
- 3.3-V I/O, 1.8-V Core
- Power Dissipation: <1 W for Four Channels
- Package: 252-Ball, 17-mm PBGA, 1-mm Pitch

APPLICATIONS

- Cellular Base Transceiver Station Transmit and Receive Channels
 - WCDMA
 - CDMA2000
- Radar
- General Filtering
- Test and Measurement

GC5016 transceiver mode

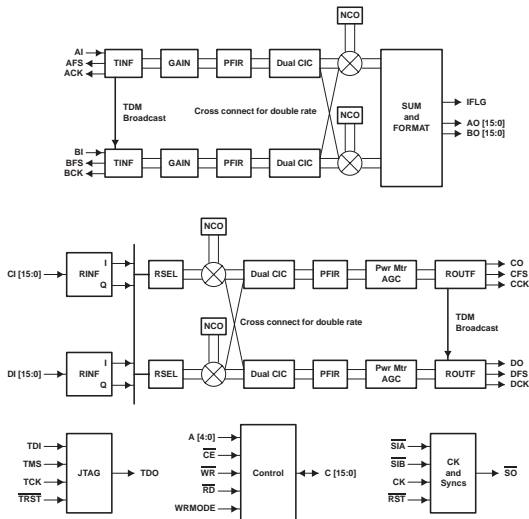


Figure 3. GC5016 in Transceiver Mode

GC5016 upconversion CIC

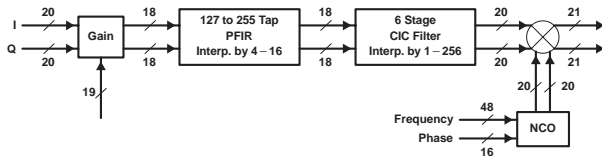


Figure 16. Up-Conversion Channel Detail

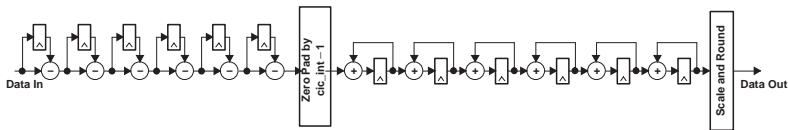


Figure 21. 6-Stage CIC Interpolate by *cic_int* Filter

GC5016's DDS

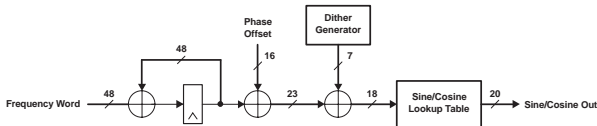


Figure 6. Numerically Control Oscillator (NCO) Circuit

The NCO's frequency, phase and accumulator can be initialized and synchronized with other channels using the *freq_sync*, *phase_sync*, and *nco_sync* controls. The *freq_sync* and *phase_sync* controls determine when new frequency and phase settings become active. Normally, these are set to always so that they take effect immediately, but can be used to synchronize frequency hopping or beam forming systems. The *nco_sync* control is usually set to never, but can be used to synchronize the NCOs of multiple channels.

The NCO's spur level is reduced to below -113 dB through the use of phase dithering. The spectrums in Figure 7 show the NCO spurs for a worst case tuning frequency with and without dithering. Dithering decreases the spur level from -105 dB to -116 dB. Dithering is turned on or off using the *dither_sync* controls. Holding *dither_sync* always on freezes the dither value, effectively turning off dither.

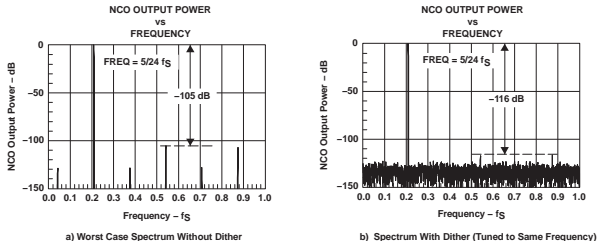


Figure 7. Example NCO Spurs With and Without Dithering

GC5016 example filter response

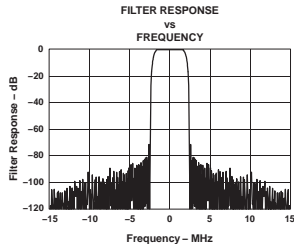


Figure 33. UMTS Filter Response

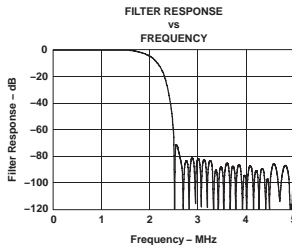


Figure 34. UMTS Filter Response Transition Region